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Babylonian Mathematical Astronomy

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Abstract

The earliest known form of mathematical astronomy of the ancient world was developed in Babylonia in the 5th century BCE. It was used for predicting a wide range of phenomena of the Moon, the Sun, and the planets. After a brief discussion of the material evidence and historical context of Babylonian mathematical astronomy, its main concepts and methods are illustrated on the basis of a tablet with computed data for Jupiter. Finally, the past, present, and future directions of research are briefly addressed.

Sources and Historical Context

All cuneiform tablets with mathematical astronomy were found in Babylon and Uruk, two major Babylonian cities and centers of learning. They were written between 380 and 48 BCE, a period covering a part of the Persian era (380–331 BCE), the reign of Alexander the Great and his dynasty (330–312 BCE), the Seleucid era (311–146 BCE), and a part of the Parthian era (145–48 BCE). The tablets from Babylon cover this entire period. Most of them were acquired by

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the British Museum (London) in the last decades of the nineteenth century after having been excavated unscientifically by local inhabitants. Subsequent excavations between 1899 and 1917 by the German archaeologist Robert Koldewey produced only about 10 astronomical tablets, including a single one with mathematical astronomy. All originate from a private house, presumably the living quarters of a scholar, located not far south of the Esagila, temple of the Babylonian supreme god Bel (Marduk) and his spouse Beltu. These are the only astronomical tablets from Babylon for which the archaeological context is reliably known, but it is generally assumed that the others also originate from private houses in the same area. Evidence from administrative tablets suggests that the astronomers who wrote them were employed by the Esagila, at least from the Achaemenid era onward. Many astronomers mentioned in these tablets belong to the Mushezib family, whose activities in Babylon can be followed over at least seven generations.

The sources from Uruk are fewer and they cease near 170 BCE, but their archaeological context is better known, because most of them came to light during scientific excavations conducted by German archaeologists between 1912 and the 1990s. One small group of early Seleucid tablets was found in the private house of the exorcist Iqisha from the Ekur-zakir family, which played a dominant role in Uruk's intellectual circles throughout the Seleucid era. Although the astronomical tablets from Iqisha's library do not mention the name of a scribe, they were likely written by him or his next kin. Another library with astronomical tablets was located in the Resh temple, sanctuary of the sky god Anu and his spouse Antu. It was discovered in a small brick-paved room adjacent to the southeastern gate of the Resh complex, some tablets still lying in their original location in niches in the walls. Most tablets in this library were written by members of the Sin-leqe-uninni family.

The beginning of Babylonian mathematical astronomy can be dated to about 400 BCE, when Babylonian astronomers invented the zodiac by dividing the path of the Sun, the Moon, and the planets into 12 equal segments of 30° and naming each one after a nearby constellation. This coordinate system became an essential ingredient of mathematical astronomy for computing positions of the Moon, the Sun, and the planets. It appears that mathematical astronomy developed rapidly after that, since the algorithms reached their final stage by about 350–310 BCE, with little evidence of further change. Most of the extant tablets were written after 220 BCE and reflect that final stage.

The corpus of mathematical astronomy consists of about 340 tabular texts, tables with numbers arranged in rows and columns, and about 110 procedure texts, verbal instructions for computing and verifying the tables. There are four kinds of tabular texts: synodic tables (numbering 230), template tables (50), daily motion tables (30), and auxiliary tables (20). In a synodic table, consecutive rows correspond to successive events of a synodic phenomenon (see below). While most planetary synodic tables contain only two columns, one for times and one for positions, the lunar tables may contain up to 21 columns, each tabulating a different astronomical quantity. Template tables contain a selection of columns corresponding to some intermediate stage in the production of a synodic table. In a daily motion table,

the zodiacal position of the Moon, the Sun, or a planet is tabulated from day to day (or from tithi to tithi, an artificial unit of time; see below). Finally, auxiliary tables provide certain numerical coefficients that are needed in the computations. About half the corpus is concerned with the Moon, the rest with the planets Mercury, Venus, Mars, Jupiter, and Saturn, apart from a few tables with daily positions of the Sun. A remarkable feature of Babylonian mathematical astronomy is the simultaneous existence of different computational systems (sets of algorithms) for predicting the same phenomena of a given planet or the Moon. These different systems form two main families known as type A and type B (see below).

Case Study: A Computed Table for Jupiter

This case study of a computed table and corresponding procedures for Jupiter system A' aims to illustrate some of the main concepts and methods of Babylonian mathematical astronomy. However, it should be kept in mind that the lunar tablets have a far higher complexity and sophistication that cannot be addressed here. The tablet consists of three joining fragments, all originating from unscientific excavations in Babylon. The largest fragment, BM 34570, is kept in the British Museum, while the others, VAT 1753 and VAT 1755, are in the Berlin museum. Their combined dimensions are $15.5 \times 14.0 \times 1.7\text{--}3.3$ cm. The obverse and most of the reverse are occupied by a synodic table which is followed by two procedures and a colophon (For a critical edition of the table see *ACT* 611; for the procedures see Ossendrijver (2012, p. 35). Figure 173.1 shows a photograph of the reverse. The table covers the years SE 180–252, of which SE 219–252 (sexagesimally: 3,39–4,12) are on the reverse. In the translation (Fig. 173.2) missing text is enclosed by square brackets and damaged text by upper corners. Since the table very likely covered five synodic phenomena (see below), about 40 % of the tablet is still missing on the right side.

Babylonian astronomers used the sexagesimal place-value notation, which means that numbers were represented as sequences of digits, each assuming a value between 0 and 59. Successive digits denote successively decreasing powers of 60, entirely analogous to our decimal system. When the astronomers took over this notation, which was invented in the Ur-III period (2000–1900 BCE) and is well known from Old Babylonian mathematical texts (1900–1600 BCE), they introduced a few minor changes, for instance, a special sign for vanishing digits (0). (However, this sign is not used for representing the number zero.) In modern translations, commas are inserted between all digits except those pertaining to 1 and 1/60, where a semicolon is used.

In order to understand the table, some basic knowledge of Jupiter's synodic cycle is necessary. The relevant synodic phenomena are the first appearance (FA), first station (S1), acronychal rising (AR), second station (S2), and last appearance (LA). FA denotes the planet's first visible rising on the eastern horizon shortly before sunrise, also known as heliacal rising, and LA its last visible setting on the western horizon shortly after sunset, also known as heliacal setting. Between LA

Fig. 173.1 Photograph of BM 34570 + VAT 1753 + 1755 (reverse)



FA	T	B	phen.	S1	T	B	phen.	AR	T	B	phen.	S2	T							
1	[3,3] ⁹	IX ¹	15	27;56,15	Sgr	appearance	3,40	I	22	17;10 ¹	Cap	station	3,40	III	24	12,22	Cap	'rising'	[3,40	..]
2	3,[40]	XI ¹	'3'	3;40	Aqr	appearance	3,41	III	11	23;10 ¹	Aqr	station	3,41	V	12	18,22	Aqr	'rising'	[3,41	..]
3	3,41	'XII	21 ¹	9;40	Psc	appearance	3,42	III	29	29;10	'Psc'	station	3,42	V	30	24,22	Psc	rising	[3,42	..]
4	3,43	I	9	15;40	'Ari	appearance	'3',43	V	17	5;10	'Tau'	station	3,43	VII	18	0,22	Tau	rising	[3,43	..]
5	3,44	II	27	21;22,30	'Tau'	[appearance]	'3',44	VII	3	9;39,212,30 ¹	Gem		3,44	IX	5	5;9,22,30	Gem	rising	'3',44	..]
6	3,45	III	13	25;7,30	Gem	appearance	3,'45	VII	18	12;55	Cnc	station	3,45	IX	20	8;54,22,30	Cnc	rising	'3',45	..]
7	3,46	IV	26	26;40	Cnc	appearance	3,46	VIII	30	12;55	Leo	'station	'3',46	XI	2	8;55	Leo	rising	'3',47	..]
8	3,47	VI	8	26;40	Leo	appearance	3,47	IX	12	12;55	Vir	'station	3,47	XI ¹	14	8;55	Vir	rising	'3',48	..]
9	3,48	VI	20	26;40	Vir	appearance	3,48	'X	214	12;55	Lib	'station'	3,48	XII	127 ¹	8;55	Lib	rising	[3,49	..]
10	3,49	VIII	3	26;40	Lib	appearance	3,49	XII	8	13;24,22,30	Sco		3,50	I	9	8;[5] ⁵	Sco	rising ¹	[3,50	..]
11	3,50	'VIII'	17	28;52,30	Sco	appearance	3,50	XII	24	17;9,22,30	Sgr	station	3,51	II	25	12;31 ⁹ ,22,30	Sgr		3,51	..]
12	3,51	X	3	2;40	Cap	appearance	3,52	II	11	22;10	Cap	'station'	3,52	IV	12	17;21 ²	Cap	rising	3,52	..]
13	3,52	XI	21	8;40	Aqr	appearance	'3',53	II	29	2'8';10	'Aqr	station'	3,53	IV	30	23;21 ²	Aqr	rising	3,53	..]
14	3,53	XII	9	14;40	Psc	appearance	3,54	IV	17	4;10 ¹	Ari	station	3,54	VI	18	29;21 ²	[Psc	rising	3,54	..]
15	3,55	I	27	20;40	Ari	appearance	3,55	VI	5	10;10	Tau	station	3,55	VIII	6	5;22	'Tau'	[rising	3,55	..]
16	3,56	II	15	26;3,45	Tau	appearance	3,56	VI	21	14;20,37,30	Gem		3,56	VIII	22 ¹	9;50,37,30 ¹	[Gem		3,56	..]
17	3,57	III	30	29;48,45	Gem	appearance	3,57	VIII	6	17;5,37,30	Cnc		3,57	X	'8'	13;5	Cnc	rising	3,57	[..]
18	3,58	IV	13	0;50	Leo	appearance	3,58	VIII	18	17;5 ¹	Leo	station	3,58	X	20	13;5	Leo	rising	'3',59	..]
19	[3,59]	V	25	0;50	Vir	appearance	3,59	IX	30	17;5 ¹	Vir	station	3,59	XII	2	13;5	Vir	rising	4,0	[..]
20	[4,0	VII]	'8'	0;50	Lib	appearance	4,0	XI	12	17;5 ¹	Lib	station	4,0	XII ¹	14	13;15	Lib	rising	[4,1	..]
21	[4,1	VII]	'20'	0;50	Sco	appearance	4,1	XI	215	18;5,37,30 ¹	Sco	station	4,2	I	26	13;35,37,30	Sco		[4,2	..]
22	[4,2	IX	4	3];33,45	Sgr	appearance	4,13	I	[1] ¹	21;50,37,30	Sgr	station	4,3	III	13	17;20,37,30	Sgr		'4',3	..]
23	[4,3	X	21	7;40	Cap'	appearance	4,4	I	'29'	27;10	Cap	station	4,4	III	30	22,22	Cap	rising	[4,4	..]
24	[4,4]	'XI	9	1'3;40	Aqr	appearance	4,5	III	17 ¹	3;10	Psc	station	4,5	V	18	28,22	Aqr	rising	[4,5	..]
25	'4,5'	XII	27	19;40	Psc	appearance	4,6	V	5	9;10	Ari	station	4,6	'VI ¹	6	4;22	Ari	'rising'	[4,6	..]
26	4,7	I	15	25;40	Ari	appearance	4,7	V	23	'15';10	Tau	station	4,7	VII	24	10,22	Tau	[rising	4,7	..]
27	4,8	III	2	0;45	Gem	appearance	4,8	VII	9	[1] ⁹ ;1,52,30	Gem	station	4,8	'IX'	10	14;31,5' ² ,30	Gem	rising	4,8	..]
28	'4,9	III	18	4;30	Cnc	appearance	4,9	VII	2'3'	[2];15	Cnc	station	4,9	IX ¹	[25	17;15	Cnc	rising	4,9	..]
29	'4,10	V	1	5	Leo	appearance	4,10	IX	5	[2];15	Leo	station'	4,10	XI	7 ¹	[17;15	Leo	rising	4,10	..]
30	'4,11'	VI	13	5	Vir	appearance	4,11	X	17	'2'[1];15	Vir	station	4,11	XII	19	'1'[7;15	Vir	rising	4,12	..]
31	From ¹ 9 Cnc until 9 Sco you add 30. (The amount) by which it exceeds 9 Sco you multiply by 1;[7,30], add to 9 Sco, and put down. From 9 Sco until 2 Cap you add 33;45. (The amount) by which it exceeds 2 Cap you multiply by 1;4, add to 2 Cap, and put down.]																			
32	From ¹ 2 Cap until 17 Tau you add 36. (The amount) by which it exceeds 17 Tau[u you multiply by 0;56],15, add to 17 Tau, and put down. From 17 Tau until 9 Cnc you add 33;45. (The amount) by which it exceeds 9 Cnc you multiply by 0;53,20, add to 9 Cnc, and put down.]																			
33	The 'days': [you compute] the distance from appearance to appearance, [and 12;5,10], this you add to it, [you add it] to your 'year' [...]																			
34	you put down the appearance of Jupiter. He who honors [...] will not [...] Tablet of [...], son of [Ea]-balassu-iqbi, descendant of Nanna-[utu] [...]																			

Fig. 173.2 Translation of BM 34570 + VAT 1753 + 1755 (reverse)

and FA, which can last several months, Jupiter cannot be observed; between FA and LA it is always visible for some portion of the night. After FA, Jupiter moves along the ecliptic in the forward direction, i.e., through the zodiacal signs in the sequence Ari, Tau, and Gem. At S1 Jupiter comes to a standstill, after which it moves in the

opposite, retrograde direction. Roughly halfway between S1 and S2, Jupiter is in opposition to the Sun. This phenomenon itself is not readily observable, but a few days earlier Jupiter's last visible so-called achroynchal rising (AR) can be observed on the eastern horizon, just after sunset. After S2, Jupiter again moves in the forward direction until it disappears at LA.

For each phenomenon there are three main columns: one for the time (modern symbol: T), one for the zodiacal position (B), and one identifying the phenomenon. In column T the date is provided as a year number of the Seleucid era (SE year 1 = 311/0 BCE) and a month name, translated here as a Roman numeral. A few words about the calendar are necessary in order to understand this column. The Babylonians used a lunisolar calendar in which the beginning of the month was determined by the first appearance of the lunar crescent, so that the length of the month varied irregularly between 29 and 30 days, the mean value being 29.53 days. Since 12 such months fall short of 1 year by about 11 days, an extra month was occasionally inserted in order to prevent the months from running out of step with the seasons, so that month I (Nisannu) always begin in March/April. In the 6th c. BCE, Babylonian astronomers introduced a highly effective 19-year intercalation cycle, whereby 1 extra month, either a VI_2 or a XII_2 , was inserted in 7 out of 19 years according to a fixed pattern that could be readily continued to arbitrary future dates. Column T also contains the date expressed in *mean tithis* (1–30) within the month. The mean tithi is an artificial unit corresponding to $1/30$ of the mean synodic month, so that 1 mean tithi is slightly shorter than 1 day. By using mean tithis instead of days, Babylonian astronomers circumvented the computation of the varying lengths of future months (29 or 30 days), which would be required if times were to be expressed as actual dates in the civil calendar. Positions are defined in terms of a zodiacal sign and a number of degrees (0–30) within that sign (From procedure texts, we know that Babylonian astronomers could also compute Jupiter's distance to the ecliptic, but this quantity is not present in this or any other of the extant tables).

The table was computed with a set of algorithms known as Jupiter system A' , one of about ten known computational systems for Jupiter. After writing down the initial values in Obv. 1, the rest of the table was filled by updating B and T from one ($i - 1$) to the next occurrence (i) of the same synodic phenomenon, in accordance with the following modern formulas:

$$B_i = B_{i-1} + \sigma, \quad (173.1)$$

$$T_i = T_{i-1} + \tau. \quad (173.2)$$

Here σ is Jupiter's displacement along the zodiac, also known as synodic arc, and τ is the corresponding synodic time. This approach, whereby the coordinates of each synodic phenomenon are updated independently of the others, is typical of Babylonian astronomy. The value of σ depends on Jupiter's position, which reflects the varying speed of Jupiter and the Sun during their apparent motion around the Earth. Two different algorithms were used for computing σ . In type B systems, σ

Table 173.1 Jupiter system A': parameters of the step function for the synodic arc

j	Zone	σ_j	r_j
1	9 ° Cnc – 9 ° Sco	30 °	1;7,30
2	9 ° Sco – 2 ° Cap	33;45 °	1;4
3	2 ° Cap – 17 ° Tau	36 °	0;56,15
4	17 ° Tau – 9 ° Cnc	33;45 °	0;53,20

varies between a minimum and a maximum with a constant difference, which results in a so-called zigzag function. In Jupiter system A', which is a type A system, σ is modeled as a step function of the zodiacal position. This means that the zodiac is divided into a number of zones, four in this case, each featuring a different constant value of σ . The corresponding rules are provided in the first procedure (Rev. 31–32), which begins as follows: if Jupiter is between 9 ° Cnc and 9 ° Sco then $\sigma = 30$ °. This rule was used, e.g., for computing the positions of FA in Rev. 8–10. Then follows a second rule which modifies the updated position if it is beyond 9 ° Sco, i.e., in the next zone: the excess of σ beyond 9 ° Sco is multiplied by the coefficient 1;7,30 and the product is added to 9 ° Sco. This so-called transition rule explains, for instance, the position of FA in Rev. 11. Analogous rules apply in the other three zones (Table 173.1). Occasional scribal errors which were not passed on to the next line (e.g., the underlined digits in Rev. 17) prove that the tablet was copied from an original in which these errors were absent.

As it turns out, the coefficients, r_j , always satisfy $r_j = \sigma_{j+1}/\sigma_j$. It can be shown that this has two important effects. First, if Jupiter crosses from one zone (j) into the next ($j + 1$), then σ assumes a value in between σ_j and σ_{j+1} . Second, Jupiter's positions satisfy a strict period relation. If one would continue this table for 391 lines, then all phenomena would repeat at exactly the same positions as in Obv. 1. During that interval, Jupiter performs exactly 36 revolutions around the zodiac, the Sun 427. This period relation, 391 repetitions = 36 revolutions = 427 year, belongs to the empirical core of Jupiter system A' which was derived from observations.

The algorithm for updating T is badly preserved on this tablet (Rev. 33–34), but other procedure texts and the synodic tables imply that the synodic time equals the following:

$$\tau = \sigma + 12 \text{ months} + 12; 5, 15 \text{ mean tithis} \quad (173.3)$$

This algorithm also reflects the empirical behavior of Jupiter. Note that the tabulated times were rounded to whole mean tithis, but it can be shown that this was done after computing the entire column to full precision – another indication that the tablet is a modified copy. To work out one example of the updating of T for FA, recall that B in Rev. 9 was obtained by adding $\sigma = 30$ ° to the previous value; hence, the corresponding τ equals 12 months +42;5,15 mean tithis. Since 30 mean tithis equal 1 month, τ can be rewritten as 13 months +12;5,15 mean tithis. In order to find the month obtained by adding this to T in Rev. 8, the 19-year intercalation scheme must be consulted, which would reveal that year SE 3,47 has 13 months.

Hence, SE 3,47 VI 8 (Rev. 8) +13 months +12;5,15 mean tithis yields SE 3,48 VI 20 (ignoring all fractions), which was written in Rev. 9.

The procedure texts, including those on the present tablet, are innovative in terms of how they represent mathematical operations, for instance, by using a notation for abstract quantities of undetermined magnitude (“variables”). Another innovation, found mainly in the lunar texts, concerns the usage of additive and subtractive numbers for quantifying corrections (Ossendrijver 2012).

The tablet concludes with a colophon mentioning [...]balassu-iqbi of the Nanna-utu family, who is either the scribe or the owner of the tablet. His full name may be Marduk-zera-ibni, son of Ea-balassu-iqbi, a known scholar of the Nanna-utu family who copied several lamentations and other religious texts during the years SE 177–178. This is consistent with the date of the tablet, since it was probably written near the initial date of the table, SE 180 (132/1 BCE). Other astronomical tablets of Marduk-zera-ibni have not been found, but the astronomical activities of the Nanna-utu family can be traced until 48 BCE, when a descendant produced the latest datable tablet with mathematical astronomy, a synodic table for the Moon (ACT 18) computed in accordance with system A.

Past, Current, and Future Directions of Research

Near 1880 Joseph Epping and Johann Strassmaier, German Jesuits, rediscovered Babylonian mathematical astronomy among tablets from Babylon that had recently arrived in the British Museum. Already two decades later, Franz Xaver Kugler, also a German Jesuit, succeeded in correctly explaining most of the algorithms of Babylonian mathematical astronomy. After this pioneering phase, it was Otto Neugebauer who, in 1935, set out to produce the first complete translation of the corpus, which appeared as *Astronomical Cuneiform Texts* (Neugebauer 1955). Near 1990 a new phase of research began with a critical evaluation of the historiography of Babylonian astronomy by F. Rochberg (2004). Common perceptions of a fundamental difference between scientific cultures in the ancient Near East and classical Greece turned out to be untenable. Furthermore, the internalist approach of previous researchers, with their focus on the reconstruction of algorithms, has made way for a more holistic one which aims to explain Babylonian astronomy in its institutional, political, religious, and social contexts. In a similar spirit, new translations of mathematical tablets (Høyrup 2002) that are more faithful to Babylonian concepts than was previously the case have revised our understanding of Babylonian mathematics. This approach also underlies a new edition of the procedure texts of Babylonian mathematical astronomy (Ossendrijver 2012).

Two important, badly understood aspects of Babylonian mathematical astronomy are its development in the formative period 400–330 BCE and its practical applications. It would be of great interest to know the steps by which the algorithms were constructed from empirical data and certain basic assumptions. Although several partial scenarios have been proposed for some of the algorithms, fully satisfying and comprehensive derivations remain to be found. The second question

has not received much attention. Some possible applications that seem obvious at first sight turn out to be unsatisfactory in one way or another. In the Seleucid era the beginning of the month was no longer established by observing the first lunar crescent, but by predicting it. However, it appears that this was not done with the algorithms of mathematical astronomy (Steele 2007). Babylonian horoscopes, which contain computed positions of the planets for the date of birth, are another possible application of mathematical astronomy, but the daily motion tables, from which these data may have been extracted, are few in number. As mentioned, mathematical astronomy is dominated by synodic tables, for which no concrete application has been identified. Both issues may be resolved by exploring more closely the connections between mathematical astronomy, the far more numerous astronomical diaries and related texts, and astrological texts. The diaries contain the empirical data from which the algorithms were derived, and they may provide clues to their applications. It is particularly striking that the diaries systematically combine astronomical data with market prices, river levels, weather phenomena, and historical events. This suggests a desire to predict these phenomena through their assumed correlations with astronomical phenomena.

Cross-References

- [Babylonian Observational and Predictive Astronomy](#)

References

- Høyrup J (2002) Lengths, widths, surfaces. A portrait of Old Babylonian algebra and its kin. Springer, New York
- Neugebauer O (1955) Astronomical cuneiform texts (=ACT). Springer, New York
- Ossendrijver M (2012) Babylonian mathematical astronomy: procedure texts. Springer, New York
- Rochberg F (2004) The heavenly writing. Divination, horoscopy, and astronomy in Mesopotamian culture. Cambridge University Press, Cambridge/New York
- Steele J (2007) The length of the month in Mesopotamian calendars of the first millennium BC. In: Steele JM (ed) Calendars and years. Astronomy and time in the ancient Near East. Oxbow, Oxford, pp 133–148